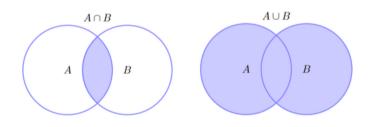
A Framework of Private Set Operations from Multi-query Reverse Private Membership Test



Yu Chen Shandong University

Talk based on the following joint works



Yu Chen, Min Zhang, Cong Zhang, Minglang Dong, Weiran Liu. Private Set Operations from Multi Query Reverse Private Membership Test. *PKC* 2024.

Outline

- PSO Framework from mqRPMT
- Construction of mqRPMT
 - 1st Construction from Commutative Weak PRF
 - 2nd Construction from Permuted Oblivious PRF
 - Connection Between mqPMT and mqRPMT
- 3 Comparison and Experimentation
- 4 Summary



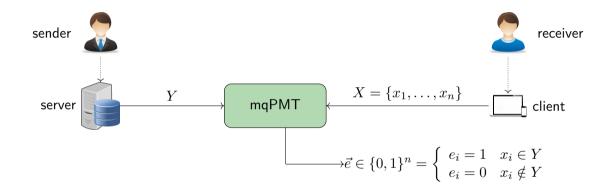
The landscape of PSO is isolated and complex. Is there a unified yet simple framework?



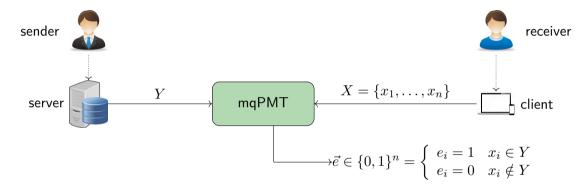
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Start Point: multi-query Private Membership Test (mqPMT) underlying PSI

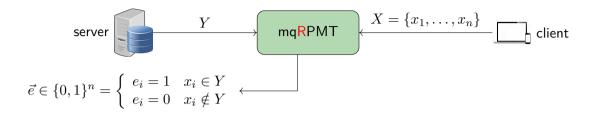


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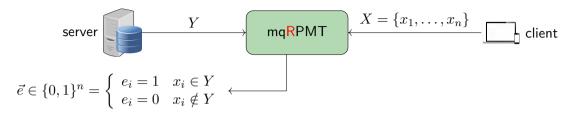


• Problem: the client learns both x_i and e_i , a.k.a. the intersection \sim not suitable for protocols that should hide intersection, such as PCSI and PSU.

The core protocol: multi-query Reverse Private Membership Test (mqRPMT)

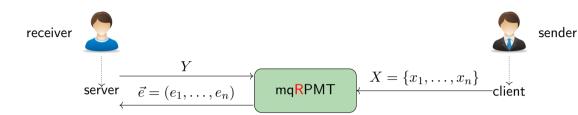


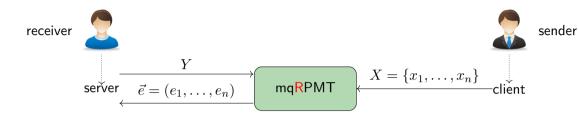
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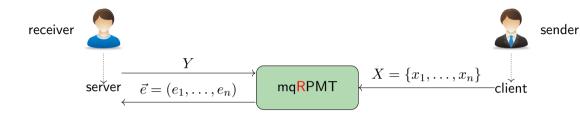
• The server learns e_i , while the client learns x_i , a.k.a. the information of intersection is shared between the two parties \sim suitable for all PSO protocols



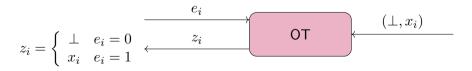


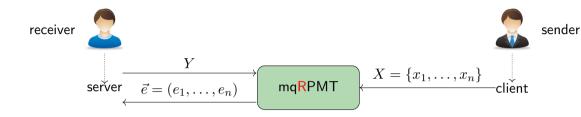


directly yields PSI-card: $|X \cap Y|$ is the Hamming weight of \vec{e}



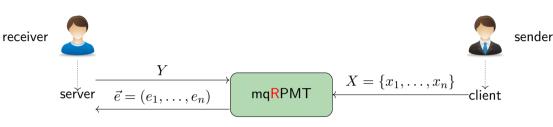
yields PSI coupled with OT: receiver obtains $X\cap Y$





yields PSU coupled with OT (flipping \vec{e}): receiver obtains X-Y

$$z_i = \left\{ \begin{array}{ccc} x_i & e_i = 0 \\ \bot & e_i = 1 \end{array} \right. \qquad \underbrace{ \begin{array}{c} 1 - e_i \\ \hline \\ z_i \end{array} } \qquad \underbrace{ \begin{array}{c} (\bot, x_i) \\ \hline \end{array} }$$

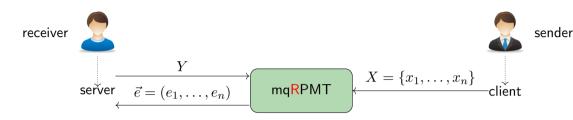


yields PSI-card-sum coupled with OT and masking trick

$$z_i = \left\{ \begin{array}{ccc} r_i & e_i = 0 & z_i \\ v_i + r_i & e_i = 1 \end{array} \right. \qquad \underbrace{ \begin{array}{c} e_i \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}} \qquad \underbrace{ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ 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receiver obtains $|X \cap Y|$

sender obtains $\sum_{x_i \in Y} v_i = \sum_{i=1}^n z_i - \sum_{i=1}^n r_i$



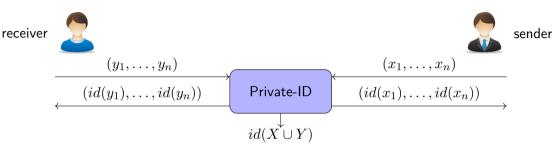
yields PSI-card-secret-share coupled with OT and masking trick

$$z_i = \left\{ \begin{array}{ccc} r_i & e_i = 0 & z_i \\ x_i \oplus r_i & e_i = 1 \end{array} \right. \qquad \text{OT} \qquad \left(\begin{array}{ccc} (r_i, x_i \oplus r_i) \\ \end{array} \right. \qquad r_i \xleftarrow{\mathbb{R}} \left\{ 0, 1 \right\}^{\ell}$$

receiver obtains $|X \cap Y|$ and z_i

sender has $x_i \oplus r_i$

Private-ID



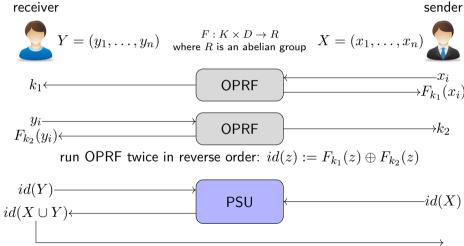
Buddhavarapu et al. [BKM⁺20] proposed private-ID:

- assigns two parties a random identifier per item
- each party obtains identifiers to his own set, as well as identifiers of the union

With private-ID, two parties can sort their private set w.r.t. a global set of identifiers, and then can proceed any desired <u>private computation item by item</u>, being assured that identical items are aligned.

Prior Construction of Private-ID

 $[{\sf BKM}^+20]$ gave a concrete DDH-based protocol. $[{\sf GMR}^+21]$ showed how to build private-ID from OPRF and PSU.



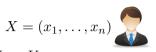
Our Construction of Private-ID

receiver



 $Y = (y_1, \dots, y_n)$

sender



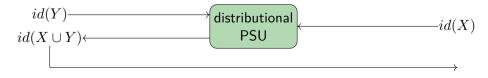
$$G: K \times D \to R$$
 where $K = K_1 \times K_2$

$$\{y_i\}_{i=1}^n \longrightarrow \{x_i\}_{i=1}^n$$

$$k_1, \{G_{k_1,k_2}(y_i)\}_{i=1}^n \longleftrightarrow k_2, \{G_{k_1,k_2}(x_i)\}_{i=1}^n$$

$$set id(z) = G_{k_1,k_2}(z)$$

standard notion are defined w.r.t. any private inputs \rightarrow arbitrary protocol composition relaxed notion w.r.t. distribution of private inputs → efficiency improvement



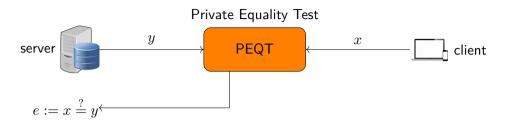
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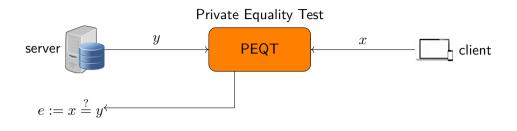
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Starting Point: PEQT



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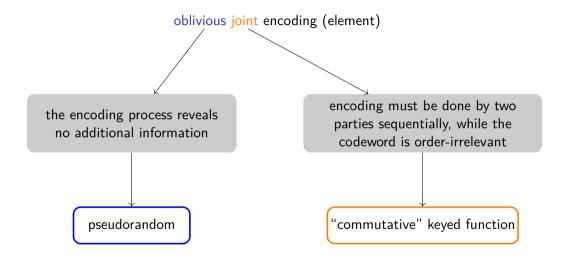


Observation: PEQT is not only an extreme case of mqPMT, but also an extreme case of mqRPMT

Goal: build PEQT amenable to extension:

$$y \sim Y = \{y_1, \dots, y_m\}, \ x \sim X = \{x_1, \dots, x_n\}, \ e \sim \vec{e} = (e_1, \dots, e_n)$$

High-level Idea



Commutative Weak PRF

We first formally define two standard properties for keyed functions.

Composable. For a family of keyed functions $F: K \times D \to R$, F is 2-composable if $R \subseteq D$ (special case R = D) $\leadsto F_{k_1}(F_{k_2}(\cdot))$ is well-defined.

Commutative. A family of composable keyed functions is commutative if:

$$\forall k_1, k_2 \in K, \forall x \in D : F_{k_1}(F_{k_2}(x)) = F_{k_2}(F_{k_1}(x))$$

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Definition 1 (Commutative Weak PRF)

 $F: K \times D \to D$ is cwPRF if it satisfies weak pseudorandomness $(k \stackrel{\mathsf{R}}{\leftarrow} K, x \stackrel{\mathsf{R}}{\leftarrow} X)$ and commutative property simultaneously. When F is a permutation, we say F is cwPRP.

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Why merely weak pseudorandomness?

Commutativity denies standard pseudorandomness. Consider the following attack:

• \mathcal{A} picks $k' \stackrel{\mathbb{R}}{\leftarrow} K$, $x \stackrel{\mathbb{R}}{\leftarrow} D$, queries the <u>real-or-random oracle</u> at point $F_{k'}(x)$ and x, receiving y' and y. \mathcal{A} then outputs '1' iff $F_{k'}(y) = y'$

$$F_{k'}(y = F_k(x)) = F_k(F_{k'}(x)) = y'$$

Construction of cwPRF

Construction (DDH-based cwPRF)

- ullet Setup (1^κ) : runs $\mathrm{GroupGen}(1^\kappa) o (\mathbb{G},g,p)$, output $pp = (\mathbb{G},g,p)$ which defines $F: \mathbb{Z}_p imes \mathbb{G} o \mathbb{G}$ as $F_k(x) := x^k$
- KeyGen(pp): outputs $k \stackrel{\mathsf{R}}{\leftarrow} \mathbb{Z}_p$.
- Eval(k,x): on input $k \in \mathbb{Z}_p$ and $x \in \mathbb{G}$, outputs x^k .

DDH assumption \Rightarrow weak pseudorandomness

Commutativity:
$$\forall k_1, k_2 \in K$$
 and $\forall x \in D$: $F_{k_1}(F_{k_2}(x)) = x^{k_1 k_2} = F_{k_2}(F_{k_1}(x))$

cwPRF is the "right" cryptographic abstraction of the classic DH function

Post-quantum Secure cwPRF

cwPRF can be analogously built from weak pseudorandom efficient group action, which is in turn based on supersingular isogeny assumption.

• Supersingular isogeny is still believed to be post-quantum secure so far, but its presumed post-quantum security is shaky.

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Can we build cwPRF from lattice-based assumption?

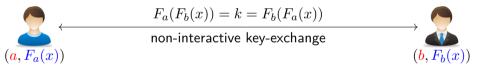
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Can we build cwPRF from lattice-based assumption?

Note that cwPRF \Rightarrow NIKE.



A recent result of Guo et al. [GKRS22] indicated that it would be difficult to construct NIKE from lattice-based assumptions.

giving lattice-based cwPRF or proving impossibility will lead to progress on some other well-studied questions in cryptography

Randomness Enhancement

But what we need for mqRPMT is standard pseudorandomness.

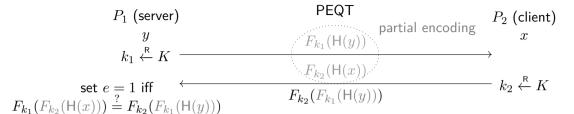
Solution: hash-then-evaluate

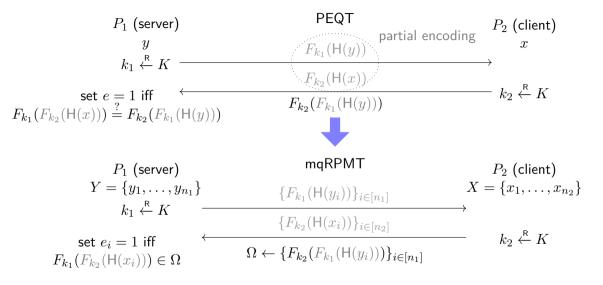
- Domain extension: handle arbitrary domain $X = \{0, 1\}^*$
- Randomness amplification: weak → standard

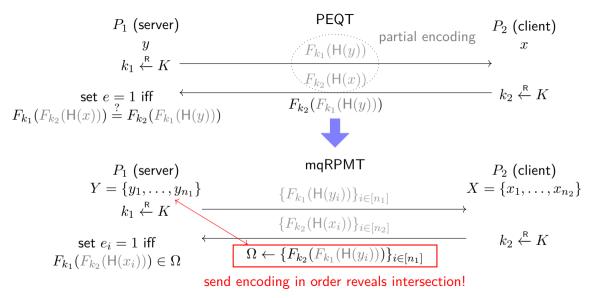
$$X \xrightarrow{\text{random oracle H}} D \xrightarrow{\text{random mess amplification}} D \xrightarrow{\text{weak PRF } F_k(\cdot)} X$$

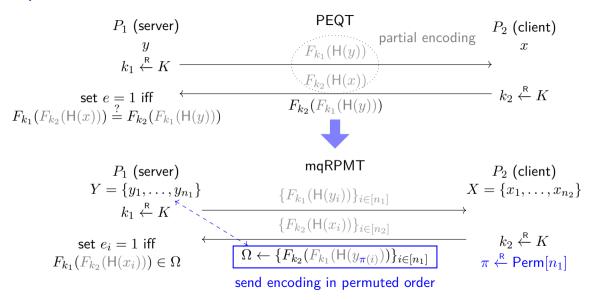
Commutativity still holds w.r.t. H (suffice for mqRPMT)

$$F_{k_1}(F_{k_2}(\mathsf{H}(x))) = F_{k_2}(F_{k_1}(\mathsf{H}(x)))$$

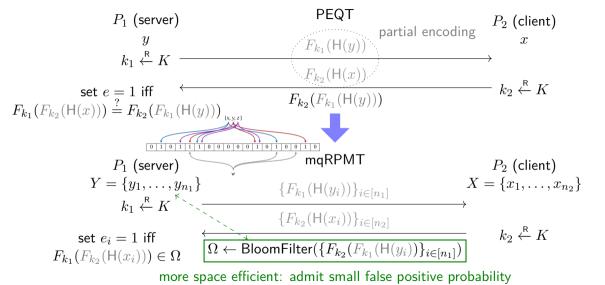








mgRPMT from cwPRF



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Complexity Analysis

Consider the balanced setting: $n_1 = n_2 = n$

Table: Complexity of cwPRF-based mqRPMT.

Computation	$4n imes F_k(\cdot) + 2n imes H(\cdot)$ hash-to-domain
Communication	$3n imes D $ or $2n imes D + n \cdot 1.44 \lambda$ ($\ll D $)

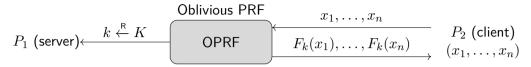
cwPRF-based mqRPMT is optimal in the sense that both computation and communication complexities are strictly linear in n

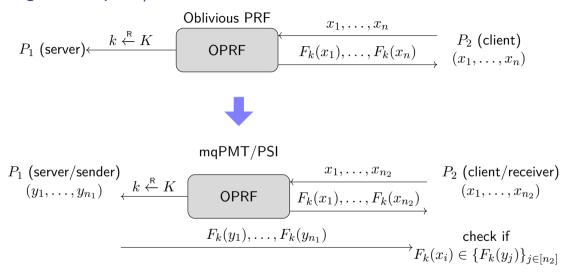
Instantiating the PSO framework with cwPRF-based mqRPMT, DDH assumption strikes back with the first strictly linear PSU protocol

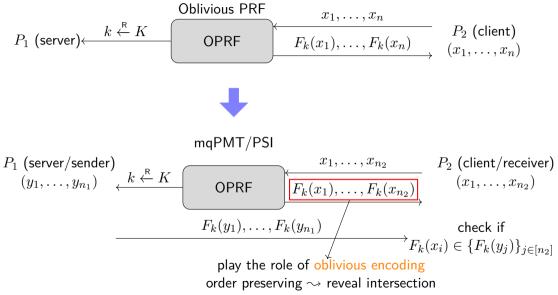
incredibly simple and efficient

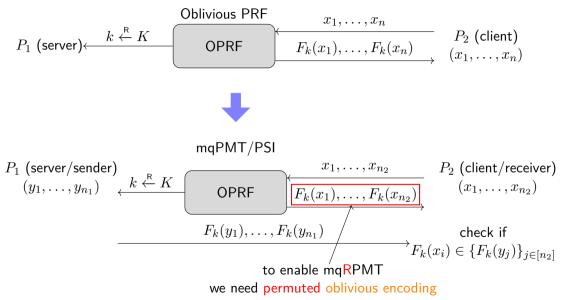
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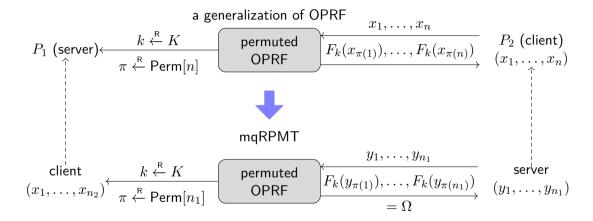




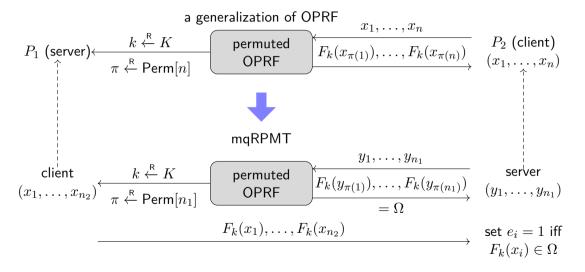
mgRPMT from Permuted OPRF



mqRPMT from Permuted OPRF

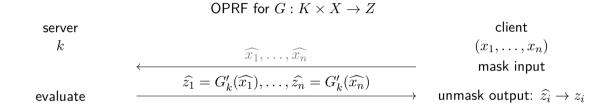


mqRPMT from Permuted OPRF



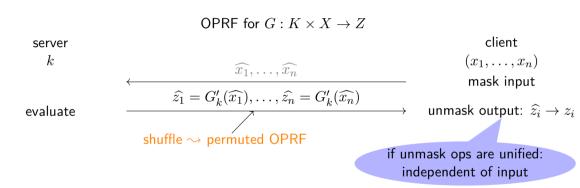
Build Permuted OPRF from cwPRP

A common approach to build OPRF is "mask-then-unmask" via homomorphism



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A common approach to build OPRF is "mask-then-unmask" via homomorphism



cwPRP enables simplest unified mask-then-unmask mask: $\hat{x} \leftarrow F_s(\mathsf{H}(x))$ evaluate: $\hat{z} \leftarrow F_k(\hat{x})$ unmask: $z \leftarrow F_s^{-1}(F_k(\hat{x})) = F_k(F_s^{-1}(\hat{x})) = F_k(\mathsf{H}(x))$

if unmask ops are unified: independent of input

Permuted OPRF from DDH-based cwPRP

Observe that the DDH-based cwPRF is actually a cwPRP $F: \mathbb{Z}_p \times \mathbb{G} \to \mathbb{G}$.

• combine $\mathsf{H}:\{0,1\}^* \to \mathbb{G} \Rightarrow \mathsf{permuted}$ OPRF protocol for $G:\mathbb{Z}_p \times \{0,1\}^* \to \mathbb{G}$ defined as $G_k(x) = F_k(\mathsf{H}(x))$.

$$\text{server} \\ k \xleftarrow{\mathbb{R}} \mathbb{Z}_p \\ \pi \xleftarrow{\mathbb{R}} \text{ Perm}[n] \\ \text{pOPRF for } G_k(x) = F_k(\mathsf{H}(x)) \\ \vdots \\ \widehat{x_1} = \mathsf{H}(x_1)^s, \dots, \widehat{x_n} = \mathsf{H}(x_n)^s \\ \vdots \\ \widehat{z_{\pi(1)}} = \widehat{x_{\pi(1)}}^k, \dots, \widehat{z_{\pi(n)}} = \widehat{x_{\pi(n)}}^k \\ \vdots \\ \widehat{z_{\pi(i)}} = \widehat{x_{\pi(i)}}^s \xrightarrow{s^{-1}} \underbrace{z_{\pi(i)} \in \widehat{z_{\pi(i)}}^s}^{\text{client}}$$

Comparison of mqRPMT from cwPRF and pOPRF

Primitive	Assumption	implied by X25519	Bloom filter optimization
cwPRF	DDH	✓	√
pOPRF	DDH	X	Х

the pOPRF-based mqRPMT is more of theoretical interest

- It can be viewed as a counterpart of OPRF-based mqPMT construction
- So far, we only know how to build pOPRF based on assumptions with nice algebra structure, but not from fast primitives such as OT or VOLE.
 - This somehow explains the efficiency gap between mqPMT and mqRPMT.

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Sigma-mqPMT

Given the efficiency gap between PSI and other PSO protocols, it is intriguing to study the connection between mqPMT and mqRPMT.

• Towards this goal, we first abstract a category of mqPMT called Sigma-mqPMT.

$$\begin{array}{c} P_1 \text{ (server)} \\ Y = (y_1, \dots, y_{n_1}) \\ a \leftarrow \mathsf{Encode}(Y) \\ \\ z_i \leftarrow \mathsf{Response}(q_i) \end{array} \xrightarrow{\begin{array}{c} P_2 \text{ (client)} \\ X = (x_1, \dots, x_{n_2}) \\ \hline \\ \vec{q} = \{q_1, \dots, q_{n_2}\} \\ \\ \vec{z} = \{z_1, \dots, z_{n_2}\} \\ \hline \\ e_i \leftarrow \mathsf{Test}(a, z_i) \end{array}$$

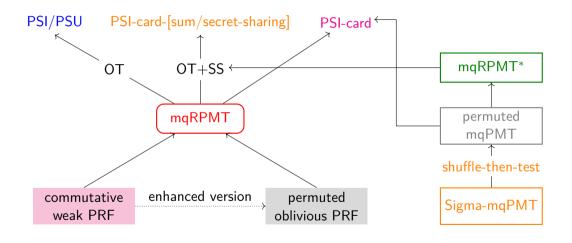
- **Reusable:** a (best interpreted as encoding of Y) can be safely reused.
- Context-independent: q_i is only related to a, x_i under test and P_2 's randomness.
- Stateless test: Test algorithm can work without knowing (x_i, q_i) .

mqRPMT* from Sigma-mqPMT

Via the "permute-then-test" approach, we can tweak Sigma-mqPMT to mqRPMT* (additionally reveal intersection size to client).

- translate a category of PSI protocols (such as [Mea86, FIPR05, CLR17]) to other PSO protocols (allowing both parties learn the intersection size).
- make the initial step towards establishing the connection between mqRPMT and mqPMT.

Summary of Main Results



Outline

- 1 PSO Framework from mqRPMT
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We implement our PSO framework via the following vein

EC groups DDH-based cwPRF \sim mqRPMT \sim PSO framework

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EC groups DDH-based cwPRF → mqRPMT → PSO framework

- NIST P-256

 ▼ (also known as secp256r1 and prime256v1)
 - hash-to-point operation is expensive \approx non-fixed Exp
 - point compression halves communication cost
 - \sim point decompression is expensive \approx non-fixed Exp

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- ② Curve25519 ★ (de facto alternative of NIST P-256)
 - numerous merits: no backdoor, fast Exp, immunity against side-channel attacks
 - ullet allow "Exp" with only X-coordinate \sim halve communication & no decompression
 - any 32-byte bit array corresponds to the X-coordinate of a valid EC point \sim hash-to-point operation is almost free

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For the first time, Curve25519 fully unleashes its power in PSO area. Correct the prejudice that "public-key operations are expensive":

 By leveraging optimized implementation, their performances are comparable with symmetric-key operations

Implementation Features



Modular design: admit flexible combination to support various scenarios



Minimum dependency: only require OpenSSL and OpenMP



Multi-platforms: run smoothly on Linux and MacOS



Rich functionality: support all PSO operations



Highly parallelizable: scalable \sim support large-scale applications

```
> oprf
                                   uint8 t k1[32]:
                                   PRG::Seed seed = PRG::SetSeed(fixed seed, 0): // initialize PRG
> peqt
                                   GenRandomBytes(seed. k1. 32): // pick a key k1
> psi
∨ pso
                                   std::vector<EC25519Point> vec Hash Y(pp.SERVER LEN):
G marpmt private id.hpp
                                   std::vector<EC25519Point> vec Fk1 Y(pp.SERVER LEN):
G mgrpmt_psi_card_sum.hpp
G marpmt psi card.hpp
                                   #pragma omp parallel for num threads(thread count)
@ marpmt_psi.hpp
                                   for(auto i = 0: i < pp.SERVER LEN: <math>i++){
@ marpmt_psu.hpp
                                        Hash::BlockToBytes(vec Y[i], vec Hash Y[i].px, 32);
@ cwprf_mgrpmt.hpp
                                        x25519 scalar mulx(vec Fk1 Y[i].px. k1. vec Hash Y[i].px):
```

Implementation Details

Dev/Test environment	Other Parameters
CPU = Intel i7 2.50 GHZ	$\kappa = 128$, $\lambda = 40$
Physical core $= 8$	item length $=128$ bits
RAM = 8GB	set sizes= $\{2^{12}, 2^{16}, 2^{20}\}$
OS = Ubuntu 20.04	LAN = 10Gbps,WAN = 50Mbps,RTT = 80ms

Protocols:

• mqRPMT, PSI, PSI-card, PSI-card-sum, PSU, Private-ID

Test items:

- Functionality
- Computation cost: total running time
- Communication cost: sum of two parties

Core protocol: mqRPMT

				Running	time (s)			Со	mm. (I	MB)
Protocol	T		LAN			WAN	total			
		2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
	1	0.50	7.20	114.16	1.39	9.68	136.27			
mqRPMT♦	2	0.31	3.89	62.09	1.14	6.54	86.60	0.52	8.35	133.6
	4	0.22	2.37	40.41	1.11	5.08	62.77			
Speedup		1.6-2.3 ×	1.9-3.0 ×	1.8-2.8 ×	1.2-1.3 ×	1.5-1.9 ×	1.6-2.2 ×	-	-	_
	1	0.50	8.00	128.00	1.35	10.15	141.52			
mqRPMT▼	2	0.32	5.05	80.69	1.18	7.11	94.19	0.27	4.35	69.6
	4	0.23	3.54	58.40	1.08	5.54	71.26			
Speedup		1.6-2.2 ×	1.6-2.3 ×	1.6-2.2 ×	1.1-1.3×	1.4-1.8 ×	1.5-2 ×	-	-	_
	1	0.26	3.51	54.85	0.81	5.41	68.68			
mqRPMT★	2	0.15	1.79	28.24	0.75	3.83	41.38	0.26	4.23	67.66
	4	0.10	1.07	15.32	0.72	3.09	28.31			
Speedup		1.7-2.6 ×	2.0-3.3 ×	1.9-3.6 ×	1.1-1.1 ×	1.4-1.8 ×	1.7-2.4 ×	ı	_	_

strict linear complexity & high parallelism

 2^{20} scale: #time $< 15 \mathrm{s}$ using 4 threads on laptop, #communication $< 70 \mathrm{M}$

PSI: Performance and Comparison

			Running	Comm. (MB)						
PSI		LAN			WAN			total		
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	
[PRTY19]*	5.51	88.64	1418.20	5.82	90.79	1498.67	0.30	4.74	76.60	
Our PSI [♦]	0.50	7.24	114.66	1.71	10.50	142.45	0.68	10.61	169.37	
Our PSI [▼]	0.55	8.04	128.18	1.73	11.02	148.18	0.42	6.61	105.23	
Our PSI★	0.29	3.56	55.11	1.19	6.38	75.56	0.41	6.48	103.31	
DH-PSI ★	0.22	3.39	54.79	0.92	5.57	69.31	0.28	4.57	74.1	

compared to existing DH-PSI implementation: # time speeds up $4.9\mbox{-}25.7\times$

			Comm. (KB)							
PSI	LAN				WAN		total			
	2^{8}	2^{9}	2^{10}	2^{8}	2^{9}	2^{10}	2^{8}	2^{9}	2^{10}	
[RT21]★	50.0	71.0	147.3	224.1	260.2	457.9	17.9	34.1	66.3	
Our PSI★	41.9	69.5	99.3	577.0	582.9	646.1	38.6	63.5	113.3	
DH-PSI★	16.49	31.80	56.91	210.42	227.33	252.32	18.48	36.68	72.8	

achieve the fastest speed in small set setting $(<2^{10})$

PSI-card: Performance and Comparison

Our framework unifies and explains prior protocols

- DDH-cwPRF-based mqRPMT: recover PSI-card [HFH99] (add Bloom filter optimization)
- DDH-pOPRF-based mqRPMT: recover PSI-card [CGT12]

			Running	Comm. (MB)						
PSI-card	LAN				WAN		total			
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	
[GMR ⁺ 21]	1.00	8.41	126.01	8.60	27.46	323.52	2.93	55.49	1030	
Our PSI-card [♦]	0.49	7.20	114.31	1.30	9.68	136.06	0.53	8.59	137.31	
Our PSI-card [▼]	0.53	8.00	128.00	1.35	10.16	141.31	0.28	4.58	73.20	
Our PSI-card★	0.27	3.51	54.89	0.82	5.42	68.31	0.27	4.46	71.30	

compared to the SOTA

time speeds up 2.3-10.5 \times , # communication reduces 11.3-15.2 \times

PSI-card-sum: Performance and Comparison

			Comm. (MB)						
PSI-card-sum		LAN			WAN		total		
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}
[IKN ⁺ 20] [▼] (deployed)	23.64	176.34	_	30.10	186.29	_	2.72	43.24	_
Our PSI-card-sum [♦]	0.51	7.22	113.66	1.46	9.68	136.27	0.65	10.12	161.40
Our PSI-card-sum [▼]	0.57	8.12	129.66	1.94	11.83	157.66	0.39	6.10	97.34
Our PSI-card-sum★	0.31	3.73	57.44	1.36	6.53	76.16	0.37	5.75	95.30



compared to the SOTA

time speeds up 22.1-76.3×, # communication reduces 7.4-7.5×

PSU: Performance and Comparison

			Running	Comm. (MB)						
PSU	LAN				WAN		total			
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	
[GMR ⁺ 21]	1.16	10.06	151.34	10.34	38.52	349.43	3.85	67.38	1155	
[?]♦	4.87	12.19	141.38	5.78	15.75	182.88	1.35	21.41	342.38	
[?]▼	5.10	15.13	187.29	5.82	17.37	210.06	0.77	12.20	195.17	
[JSZ ⁺ 22]	2.29	8.50	516.04	5.33	27.00	736.30	3.59	70.37	1341.55	
Our PSU [♦]	0.52	7.27	114.44	1.70	10.56	143.29	0.69	10.61	169.37	
Our PSU [▼]	0.57	8.04	128.20	1.76	10.92	148.15	0.42	6.61	105.23	
Our PSU*	0.30	3.55	55.48	1.19	6.38	74.96	0.41	6.48	103.31	

compared to the SOTA: first achieves strict linear complexity # time speeds up $2.4-17\times$, # communication reduces $2\times$

Private-ID: Performance and Comparison

			Comm. (MB)							
Private-ID		LAN			WAN		total			
	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	2^{12}	2^{16}	2^{20}	
[GMR ⁺ 21]	1.65	11.023	158.76	13.82	43.00	385.12	4.43	76.57	1293	
[BKM ⁺ 20]★	2.21	37.56	671.75	7.98	46.97	710.94	1.00	15.97	226.70	
Our Private-ID♦	0.55	7.28	115.63	5.34	14.83	163.43	3.12	16.91	237.55	
Our Private-ID▼	0.65	8.43	134.16	5.69	15.68	169.05	2.85	12.91	173.50	
Our Private-ID★	0.34	3.78	59.76	5.04	10.87	94.89	2.82	12.74	171.54	

- distributed OPRF: SOTA OPRF [RR22] built from VOLE and improved OKVS
- PSU protocol: cwPRF-based mqRPMT

compared to the SOTA

time speeds up $2.7\text{-}4.9\times$, # communication is slightly larger

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Summary of This Work

Unified PSO framework from mgRPMT

- show mqRPMT is complete for all PSO protocols
- greatly reduce the deployment and maintaining costs of PSO

Generic construction of mqRPMT

- cwPRF: demonstrate that DDH assumption is truly a golden goose
- permuted OPRF: make the concept of OPRF more useful; somewhat explain inefficiency of PSU/PCSI
- mqRPMT* from Sigma-mqPMT: an initial step towards the connection to mqPMT

Efficient implementation

- identify expensive ECC operations in cheap disguise
- find the perfect match: Curve25519

About Research

From [Grothendieck], I have learned not to take glory in the difficulty of a proof.

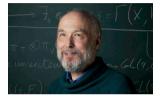


Figure: Pierre Deligne

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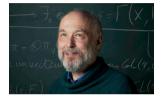


Figure: Pierre Deligne

Likewise, we do not take shame in the simplicity of our construction :-)

Simple is elegant and extremely efficient.



Thanks for Your Attention!

Any Questions?

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