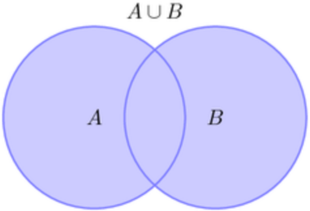
Linear Private Set Union from Multi-Query Reverse Private Membership Test



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Tutorial based on the following joint work

Cong Zhang, **Yu Chen**, Weiran Liu, Min Zhang, Dongdai Lin
Linear Private Set Union from Multi-Query Reverse Private Membership Test
USENIX Security 2023

Outline

1. Background
2. Starting Point: KRTW Protocol
3. Generic Construction of PSU
4. Two Instantiations of Generic Framework
5. Improvement and Optimization
6. Performance

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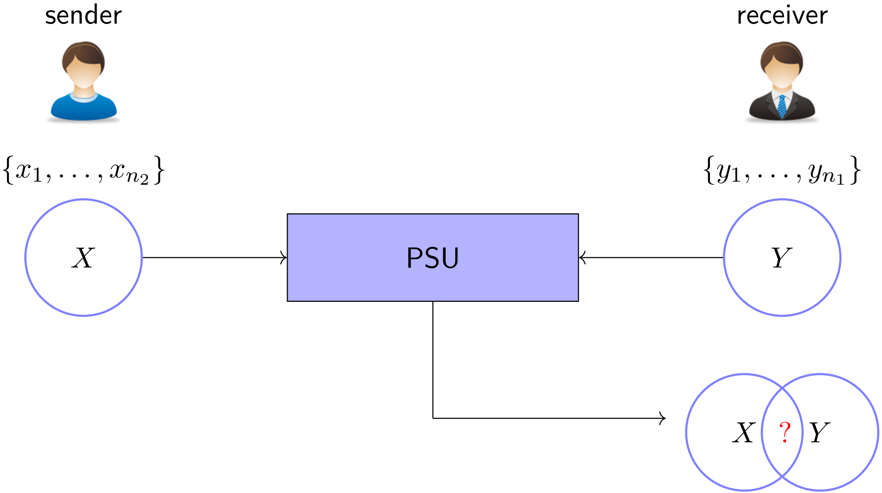
Private Set Union



In this work, we focus on the balanced setting, i.e., $n_1 \approx n_2$. For simplicity, we assume $n_1 = n = n_2$ hereafter.

Applications of PSU

PSU has found numerous applications, which include but not limit to:

- information security risk assessment [LV04]
- IP blacklist and vulnerability data aggregation [HLS+16]
- joint graph computation [BS05]
- distributed network monitoring [KS05]
- building block for private DB supporting full join [KRTW19]
- private-ID [GMR+21]

Previous Work

According to the underlying techniques, existing PSU protocols can be divided into two categories:

- Public-key techniques (e.g. AHE) [KS05, Fri07, DC17]
  - Pros. good asympotic complexity: "almost" linear computation/communication complexity
  - Cons. poor concrete efficiency: $O(\lambda)$ AHE operations per set element
- Symmetric-key techniques coupled with OT [KRTW19, GMR$^+$21, JSZ$^+$22]
  - Pros. (i) good concrete efficiency: running time is several orders of magnitude faster than AHE-based protocols;
  - Cons. poor asympotic complexity: communication/computation complexity are superlinear

Protocols based on symmetric-key techniques are plausibly quantum secure.

Motivation

Can we attain the-best-of-two-worlds: designing PSU protocols with optimal linear complexity and good concrete efficiency?

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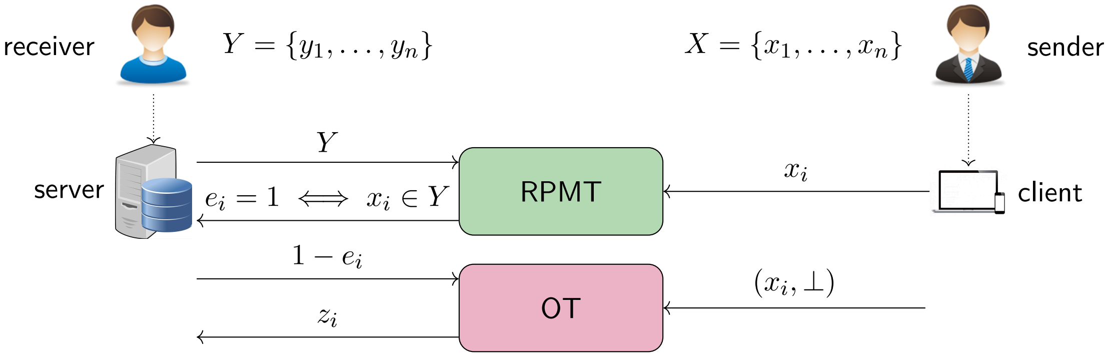
Review of KRTW (Kolesnikov-Rosulek-Trieu-Wang) Protocol



receiver $Y = \{y_1, \ldots, y_n\}$

$X = \{x_1, \ldots, x_n\}$ sender

server

Y

$e_i = 1 \iff x_i \in Y$

RPMT

x_i

client

$1 - e_i$

z_i

OT

$(x_i, \perp)$

$$z_i = \begin{cases} x_i & e_i = 0 \\ \perp & e_i = 1 \end{cases}$$

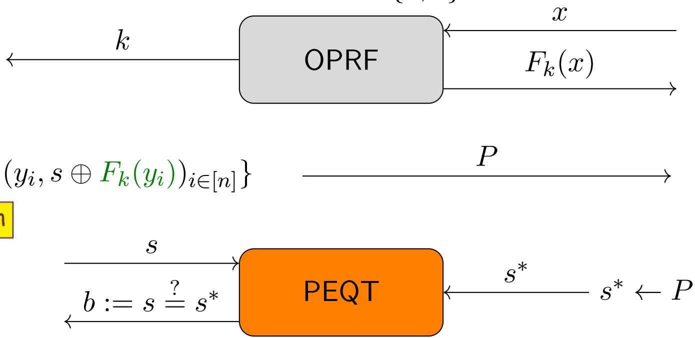
repeat the 1-vs-many PSU n times independently

Zoom In of the Sub-protocol RPMT

server $Y = \{y_1, \ldots, y_n\}$

$X = \{x_1, \ldots, x_n\}$ client

$F : K \times D \to \{0,1\}^\ell$



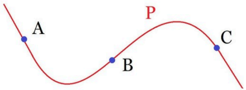
indication string of Y

$s \xleftarrow{R} \mathbb{F}_{2^\ell}$

$P \leftarrow \text{Interpolate}\{(y_i, s \oplus F_k(y_i))_{i \in [n]}\}$

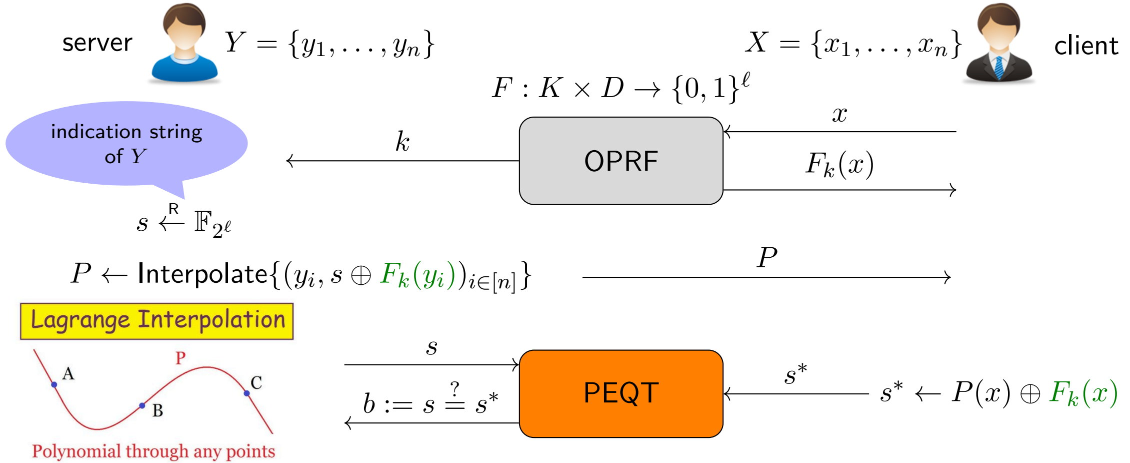
$s^* \leftarrow P(x) \oplus F_k(x)$

Lagrange Interpolation



Polynomial through any points

Zoom In of the Sub-protocol RPMT



server $Y = \{y_1, \dots, y_n\}$

$X = \{x_1, \dots, x_n\}$ client

$F : K \times D \to \{0,1\}^\ell$

indication string of Y

$s \overset{R}{\leftarrow} \mathbb{F}_{2^\ell}$

k

OPRF

x

$F_k(x)$

$P \leftarrow \text{Interpolate}\{(y_i, s \oplus F_k(y_i))_{i \in [n]}\}$

P

Lagrange Interpolation

s

$b := s \overset{?}{=} s^*$

PEQT

s^*

$s^* \leftarrow P(x) \oplus F_k(x)$

Usage of OPRF. Without OPRF masking, if $x_i \in Y$ client may learn s by evaluating $P(x_i) \rightsquigarrow$ client learns all $y_i \in Y$

Correctness and Security Analysis

Correctness. Consider the following two cases:

- If $x \in Y \Rightarrow s^* = \underline{P(y_i)} \oplus F_k(y_i) = \underline{s \oplus F_k(y_i)} \oplus F_k(y_i) = s$.
- If $x \notin Y \Rightarrow F_k(x)$ is pseudorandom. Via real-or-random argument, we conclude that for a tuple of PPT (server, client), $\Pr[s^* = F_k(x) \oplus P(x) = s] \leq 1/2^\ell$ in computational sense.

$$x \in Y \iff s = s^*$$

Security. Follows the semi-honest security of OPRF and PEQT.

Complexity Analysis

OPRF and PEQT are fast cryptographic protocols

The computation bottleneck lies at polynomial interpolation of *arbitrary* n points

- trivial algorithm using Langrange formula requires $O(n^2)$
- fast algorithm using FFT requires $O(n \log^2 n)$

The communication bottleneck lies at the representation of degree-n polynominal

- $O(n)$ field elements in $\mathbb{F}_{2^\ell}$

In sum, KRTW protocol has $O(n^2 \log^2 n)$ computation complexity and $O(n^2)$ communication complexity[1]

[1] In [KRTW19], hash-to-bin technique was used to reduce complexity. However, Jia et al. [JSZ$^+$22] pin-pointed that the improved protocol is not secure.

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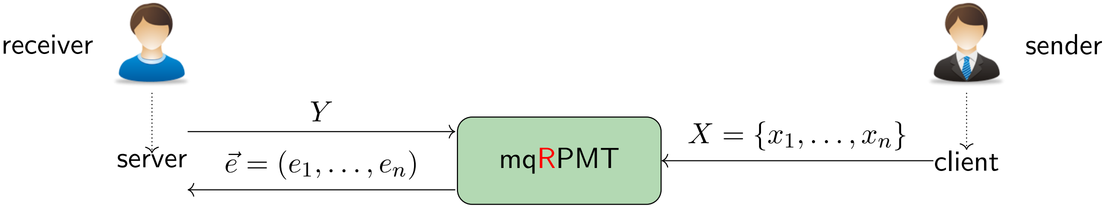
PSU from mqRPMT



receiver

server

sender

client

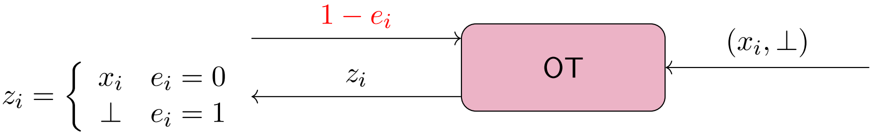
Y

$\vec{e} = (e_1, \ldots, e_n)$

mqRPMT

$X = \{x_1, \ldots, x_n\}$

yields PSU coupled with OT (flipping $\vec{e}$): receiver obtains $X \backslash Y$



$1 - e_i$

z_i

OT

$(x_i, \perp)$

$$z_i = \begin{cases} x_i & e_i = 0 \\ \perp & e_i = 1 \end{cases}$$

Essence of mqRPMT

The extension from RPMT $\rightsquigarrow$ mqRPMT is natural.

- Crux: find a batchable construction of mqRPMT achiving optimal linear complexity

How to Batch mqRPMT to Build Efficient mqRPMT

Root of inefficiency for KRTW protocol

1. degree-n polynominal interpolation is heavy
2. have to repeat polynominal interpolation n times, while batch the basic RPMT protocol is not trivial:
    - client learns the purported indication string s^* in clear $\Rightarrow$ direct reusing P let client be able to decide if $\underline{x_i \in Y \land x_j \in Y}$ by computing and compairing s^* $\rightsquigarrow$ compromise server's privacy

How to Batch mqRPMT to Build Efficient mqRPMT

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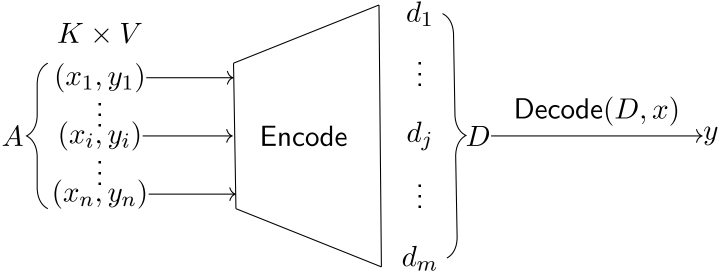
Our idea is based on two key observations.

- 1st Observation. Polynomial interpolation plays the role of oblivious key-value store.
- 2nd Observation. The usage of OPRF is three-fold:
  - server uses OPRF to derive n pseudorandom one-time pads, then encrypts the same s^* into n ciphertexts under these one-time pads.
  - client uses OPRF to decrypt a ciphertext obliviously.
  - OPRF infuses polynomial interpolation with randomness to ensure the correctness.

Oblivious Key-Value Store



rate n/m
1 is optimal

Correctness. For any $A = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ and any $x_i \in \{x_1, \ldots, x_n\}$:

$$\Pr[\text{Decode}(D, x_i) = y_i] \geq 1 - \text{negl}(\lambda), \text{ where } D \leftarrow \text{Encode}(A).$$

Randomness. For any $A = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ and any $x \notin \{x_1, \ldots, x_n\}$:

$$\text{Decode}(D, x) \approx_s U_V, \text{ where } D \leftarrow \text{Encode}(A).$$

Obliviousness. For any $(x_1^0, \ldots, x_n^0) \neq (x_1^1, \ldots, x_n^1)$:

$$\text{Encode}((x_1^0, y_1), \ldots, (x_n^0, y_n)) \approx_c \text{Encode}((x_1^1, y_1), \ldots, (x_n^1, y_n)), \text{ where } y_i \overset{R}{\leftarrow} V.$$

OKVS Off-the-Shelf

Table: Comparison of Different OKVS

scheme	rate	encoding	decoding	randomness	obliviousness
Interpolation Polynomial	1	$O(n \log^2 n)$	$O(\log n)$	✗	✓
Garbled Bloom Filter [DCW13]	$O(1/\lambda)$	$O(\lambda n)$	$O(\lambda)$	✓	✓
Garbled Cuckoo Table [PRTY20]	0.4	$O(\lambda n)$	$O(\lambda)$	✓	✓
3H-GCT [GPR+21]	0.81	$O(\lambda n)$	$O(\lambda)$	✓	✓
RR22 [RR22]	0.81	$O(\lambda n)$	$O(\lambda)$	✓	✓
RB-OKVS [BPSY23]	0.97	$O(\lambda n)$	$O(\lambda)$	✓	✓

n is # [key-value pairs]. λ is the statistical security parameter (e.g. $\lambda = 40$).

Drop-in replacement of polynomial interpolation with better OKVS will improve efficiency immediately.

How to Batch?

Rough idea to bypass the root of efficiency

- switch the role of decryption: let server decrypt ciphertexts then match the results with the indication string.

The idea is problematic since it is insecure even against a semi-honest server.

- server records the correspondence between y_i and OKVS(y_i) $\rightsquigarrow$ server learns client's private input x by simple look-up when $x \in Y$, rather than merely the fact that $x \in Y$.

We overcome this difficulty in two steps:

1. re-factor the functionality of OPRF to encryption and oblivious decryption functionality.
2. merge the oblivious decryption functionality and PEQT into a new functionality called vector oblivious decryption-then-matching (VODM) functionality.

Encryption Scheme

SKE/PKE scheme consists of three PPT algorithms:

- KeyGen(1^κ): output a secret key k or a keypair (pk, sk).
- Encrypt$(pk/k, m)$: output a ciphertext c of m.
- Decrypt$(sk/k, c)$: decrypt c to recover m.

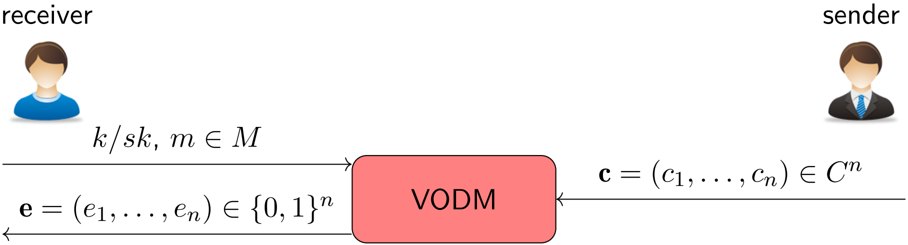
Single-message multi-ciphertext pseudorandomness. For any PPT $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, its advantage is negl$(\kappa)$.

$$\mathsf{Adv}_{\mathcal{A}}(\kappa) = \Pr\left[\beta = \beta' : \begin{array}{l} k/(pk, sk) \leftarrow \mathsf{KeyGen}(1^\kappa); \\ (m, state) \leftarrow \mathcal{A}_1(\kappa/pk); \\ \beta \leftarrow \{0,1\}; \\ c^*_{i,0} \leftarrow \mathsf{Encrypt}(k/pk, m), c^*_{i,1} \leftarrow C,\ \text{ for } i \in [n]; \\ \beta' \leftarrow \mathcal{A}_2(state, \{c^*_{i,\beta}\}_{i \in [n]}) \end{array}\right] - \frac{1}{2}$$

- Single-message multi-ciphertext pseudorandomness is a mild property satisfied by most IND-CPA secure SKE/PKE, such as PRF-based SKE, ElGamal PKE based on DDH and Regev's PKE based on LWE.

Vector Oblivious Decrypt-then-Match (VODM)

VODM w.r.t. encryption scheme $(\mathsf{KeyGen}, \mathsf{Encrypt}, \mathsf{Decrypt})$ is defined as below:



receiver

sender

$k/sk,\ m \in M$

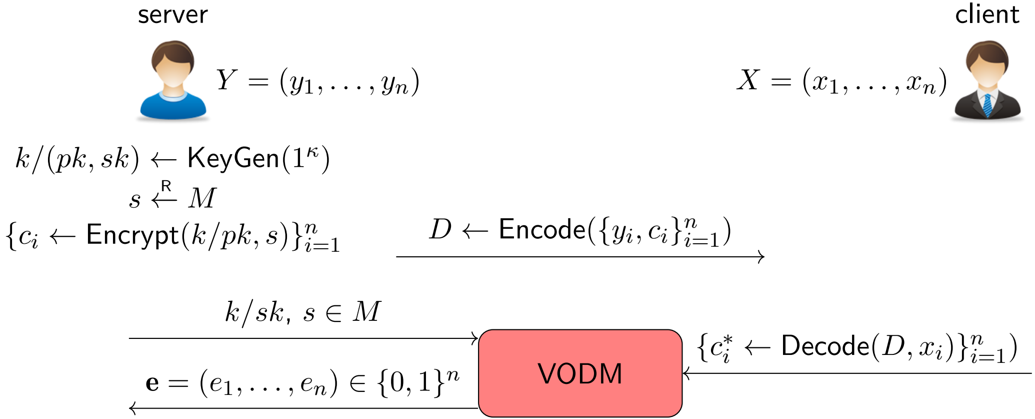
VODM

$\mathbf{c} = (c_1, \ldots, c_n) \in C^n$

$\mathbf{e} = (e_1, \ldots, e_n) \in \{0,1\}^n$

$$e_i = \begin{cases} 1 & \text{if } \mathsf{Decrypt}(k/sk, c_i) = m \\ 0 & \text{if } \mathsf{Decrypt}(k/sk, c_i) \neq m \end{cases}$$

mqRPMT from OKVS+Encryption+VODM



server

$Y = (y_1, \dots, y_n)$

client

$X = (x_1, \dots, x_n)$

$k/(pk, sk) \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \xleftarrow{\mathsf{R}} M$

$\{c_i \leftarrow \mathsf{Encrypt}(k/pk, s)\}_{i=1}^n$

$D \leftarrow \mathsf{Encode}(\{y_i, c_i\}_{i=1}^n)$

$k/sk,\ s \in M$

VODM

$\{c_i^* \leftarrow \mathsf{Decode}(D, x_i)\}_{i=1}^n)$

$\mathbf{e} = (e_1, \dots, e_n) \in \{0,1\}^n$

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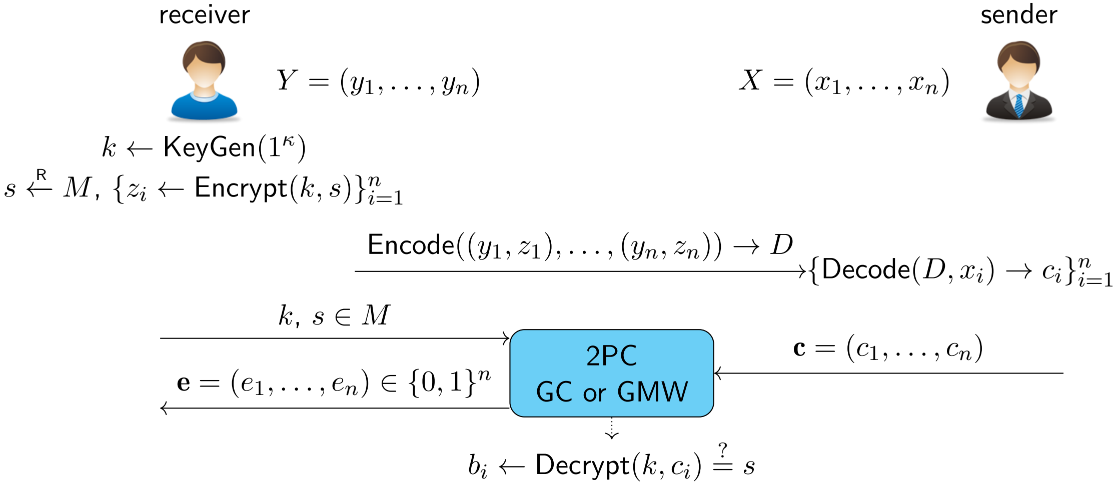
Our Focus

Choose/Design appropriate primitives to realize the framework.

1. OKVS: any off-the-shelf OKVS is fine.
2. Encryption scheme: the ones satisfy single-message multi ciphertext pseudorandomness.
3. VODM: design w.r.t. the chosen encryption scheme

We only need to foucs on step 2 and 3.

mqRPMT from SKE and Generic 2PC



receiver

$Y = (y_1, \ldots, y_n)$

sender

$X = (x_1, \ldots, x_n)$

$k \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \xleftarrow{\mathsf{R}} M, \ \{z_i \leftarrow \mathsf{Encrypt}(k, s)\}_{i=1}^n$

$\mathsf{Encode}((y_1, z_1), \ldots, (y_n, z_n)) \rightarrow D$

$\{\mathsf{Decode}(D, x_i) \rightarrow c_i\}_{i=1}^n$

$k, \ s \in M$

$\mathbf{c} = (c_1, \ldots, c_n)$

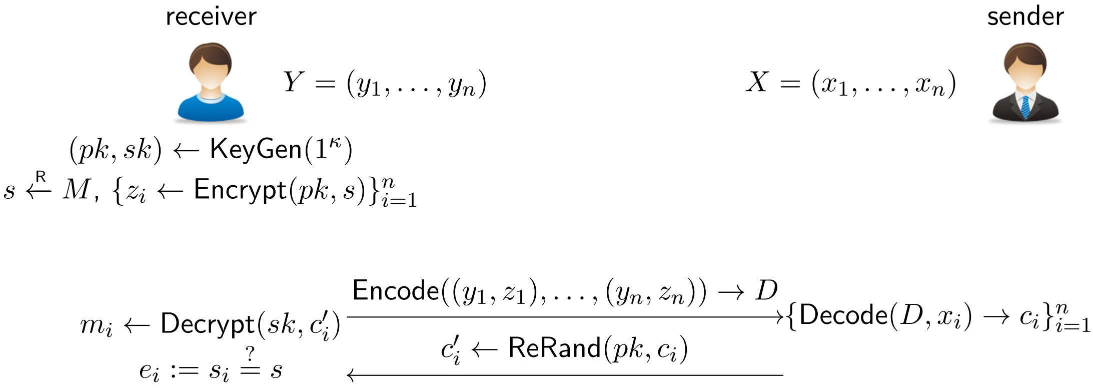
2PC
GC or GMW

$\mathbf{e} = (e_1, \ldots, e_n) \in \{0, 1\}^n$

$b_i \leftarrow \mathsf{Decrypt}(k, c_i) \overset{?}{=} s$

- SKE: choose LowMC for small circuit size
- generic 2PC: choose garbled circuit or GMW

mqRPMT from Rerandomizable PKE



receiver

$Y = (y_1, \dots, y_n)$

sender

$X = (x_1, \dots, x_n)$

$(pk, sk) \leftarrow \text{KeyGen}(1^\kappa)$

$s \xleftarrow{R} M$, $\{z_i \leftarrow \text{Encrypt}(pk, s)\}_{i=1}^n$

$\text{Encode}((y_1, z_1), \dots, (y_n, z_n)) \rightarrow D$

$\{\text{Decode}(D, x_i) \rightarrow c_i\}_{i=1}^n$

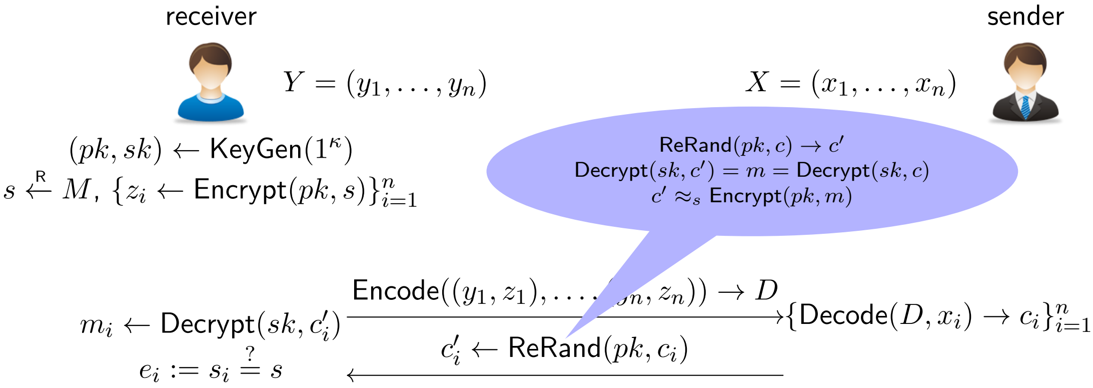
$c'_i \leftarrow \text{ReRand}(pk, c_i)$

$m_i \leftarrow \text{Decrypt}(sk, c'_i)$

$e_i := s_i \stackrel{?}{=} s$

- re-randomizable PKE: exponential ElGamal, Regev's PKE

mqRPMT from Rerandomizable PKE



receiver

$Y = (y_1, \ldots, y_n)$

sender

$X = (x_1, \ldots, x_n)$

$(pk, sk) \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \xleftarrow{\mathsf{R}} M$, $\{z_i \leftarrow \mathsf{Encrypt}(pk, s)\}_{i=1}^n$

$\mathsf{ReRand}(pk, c) \to c'$

$\mathsf{Decrypt}(sk, c') = m = \mathsf{Decrypt}(sk, c)$

$c' \approx_s \mathsf{Encrypt}(pk, m)$

$\mathsf{Encode}((y_1, z_1), \ldots, (y_n, z_n)) \to D$

$\{\mathsf{Decode}(D, x_i) \to c_i\}_{i=1}^n$

$c_i' \leftarrow \mathsf{ReRand}(pk, c_i)$

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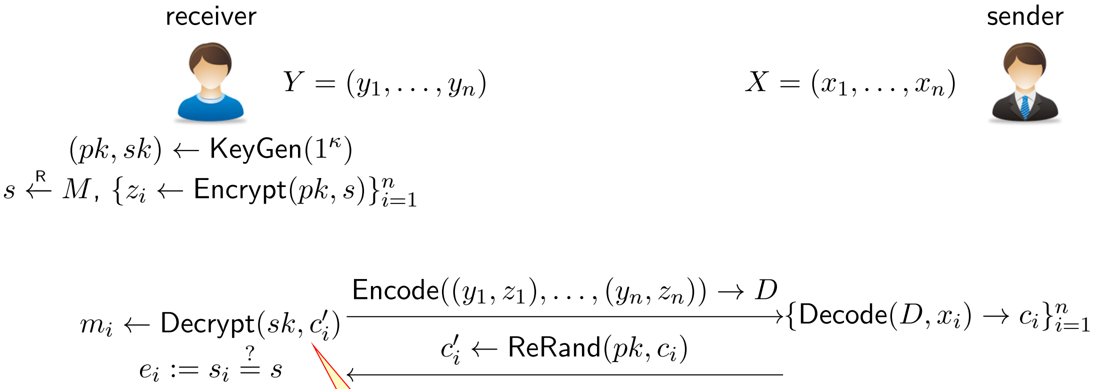

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$Y = (y_1, \ldots, y_n)$

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$(pk, sk) \leftarrow \mathsf{KeyGen}(1^\kappa)$

$s \xleftarrow{\mathsf{R}} M$, $\{z_i \leftarrow \mathsf{Encrypt}(pk, s)\}_{i=1}^{n}$

$$m_i \leftarrow \mathsf{Decrypt}(sk, c_i') \xrightarrow{\mathsf{Encode}((y_1, z_1), \ldots, (y_n, z_n)) \to D} \{\mathsf{Decode}(D, x_i) \to c_i\}_{i=1}^{n}$$

$$e_i := s_i \overset{?}{=} s \xleftarrow{c_i' \leftarrow \mathsf{ReRand}(pk, c_i)}$$

c_i' s.t. $e_i = 1$ does not leak information since receiver knows s

c_i' s.t. $e_i = 0$ does leak extra information

but such leakage is not harmful for PSU since receiver eventually learns $x_i \notin Y$

- re-randomizable PKE: exponential ElGamal, Regev's PKE

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Retrospect of the Generic Framework

Previous two mqRPMT instantiations achieve linear complexity and enjoy good concrete efficiency.

Can we further generalize the framework?
Can we improve the efficiency of concrete instantiations?

High level idea underlying our mqRPMT design

1. receiver creates a membership relation R for his set Y s.t. $\mathsf{R}(x) = 1 \iff x \in Y$.
2. receiver encrypts elements in Y w.r.t. R and sends the "encoding" of resulting ciphertexts to the sender.
3. sender is able to retrieve the ciphertext of his elements.
4. perform oblivious decrypt-then-match

We realize the right encryption scheme needed is membership encryption (ME).

- ME for set X encrypts an element x into a ciphertext, which decrypts to "1" if $x \in X$ and to "0" (intuitively).

Membership Encryption

Definition 1 (Membership Encryption (Symmetric ME))

ME for set X consists of three PPT algorithms (with X as an implicit input):

- KeyGen(1^κ): outputs a key k.
- Enc(k, x): on input a key k and an element $x \in X$, outputs a ciphertext $c \in C$. For uttermost generality, the behavior of Enc on $x \notin X$ is unspecified.
- Dec(k, c): outputs "1" indicates c is an encryption of some $x \in X$ and "0" if not.

Correctness. $\forall x \in X$, $\Pr[\mathsf{Dec}(k, c = \mathsf{Enc}(k, x)) = 1] = 1$, $k \leftarrow \mathsf{KeyGen}(1^\kappa)$.

Consistency. $\forall x \notin X$, $\Pr[\mathsf{Dec}(k, c) = 0] \geq 1 - \varepsilon(\kappa)$: $k \leftarrow \mathsf{KeyGen}(1^\kappa)$, $c \xleftarrow{\mathsf{R}} C$.

Multi-element pseudorandomness. $\forall$ distinct $x_1, \ldots, x_n \in X$

$$\{\mathsf{Enc}(k, x_i)\}_{i \in [n]} \approx_c U_{C^n},\ k \leftarrow \mathsf{KeyGen}(1^\kappa)$$

Symmetric ME naturally extends to the public-key setting:

- KeyGen outputs (pk, sk), in which pk is used to encrypt and sk is used to decrypt.

Generic Construction of ME

The essence of ME is to encrypt element's membership relation, rather than the element itself.

- Membership relation can be created by designing a mapping H from elements to X. Basically, there are two extreme cases of mapping.
    - lossy mapping: select a single indication string s as the characteristic of X, then map all elements to s, i.e., H : $x_i \to s$.
    - injective mapping: select n indication strings s_i as the characteristic of X, then map elements to distinct indication strings, i.e., H : $x_i \to s_i$.

We then present various constructions of ME by mixing encryption schemes and membership mapping.

ME from Probabilistic Encryption and Lossy Mapping

ME from probabilistic SKE and lossy mapping.

- KeyGen(1^κ): runs SKE.KeyGen(1^κ) $\rightarrow k_{\mathsf{ske}}$, picks $s \xleftarrow{\mathsf{R}} M$, sets $\mathsf{H} : X \rightarrow s$, outputs $k = (k_{\mathsf{ske}}, \mathsf{H})$.
- Enc(k, x): parses $k = (k_{\mathsf{ske}}, \mathsf{H})$, outputs $c \leftarrow$ SKE.Enc($k_{\mathsf{ske}}, \mathsf{H}(x)$).
- Dec(k, c): parses $k = (k_{\mathsf{ske}}, \mathsf{H})$, outputs '1' iff SKE.Dec(k_{ske}, c) $= s$.

ME from probabilistic PKE and lossy mapping.

- KeyGen(1^κ): runs PKE.KeyGen(1^κ) $\rightarrow (pk_{\mathsf{pke}}, sk_{\mathsf{ske}})$, picks $s \xleftarrow{\mathsf{R}} M$, sets $\mathsf{H} : X \rightarrow s$, outputs $pk = pk_{\mathsf{pke}}$ and $sk = (sk_{\mathsf{pke}}, \mathsf{H})$.
- Enc(pk, x): parses $pk = pk_{\mathsf{pke}}$, outputs $c \leftarrow$ PKE.Enc($pk_{\mathsf{pke}}, \mathsf{H}(x)$).
- Dec(sk, c): parses $sk = (sk_{\mathsf{pke}}, \mathsf{H})$, outputs '1' iff PKE.Dec(sk_{pke}, c) $= s$.

Lemma: If SKE/PKE satisfies single-message multi-ciphertext pseudorandomness, then the ME construction satisfies multi-element pseudorandomness with consistency error $1/|M|$.

Discussion

The above ME constructions are exactly the backbones of two instantiations of mqRPMT.

ME requires multi-element pseudorandomness

- the use of lossy mapping inherently stipulates that the accompanying encryption schemes must be probabilistic to satisfy single-message multi-ciphertext pseudorandomness
  $\rightsquigarrow$ ciphertext expansion is unavoidable
  $\Rightarrow$ the size of OKVS increases

Observation: if adopting injective mapping, then ME can be built from deterministic encryption schemes satisfying multi-message multi-ciphertext pseudorandomness.

ME from Deterministic Encryption and Injective Mapping

ME from deterministic SKE and injective mapping.

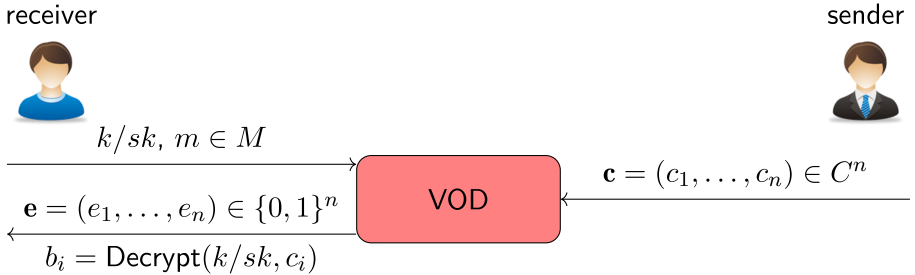
- KeyGen(1^κ): picks $k_{\mathsf{ske}} \xleftarrow{\mathsf{R}} K$, picks $s \xleftarrow{\mathsf{R}} M$, sets H : $x_i \rightarrow i$, outputs $sk = (k_{\mathsf{ske}}, \mathsf{H})$.
- Enc(k, x): outputs $c \leftarrow$ SKE.Enc$(k_{\mathsf{ske}}, \mathsf{H}(x))$.
- Dec(sk, c): outputs '1' iff SKE.Dec$(k_{\mathsf{ske}}, c) \in [n]$, where $n = |X|$.

Lemma: If SKE/PKE satisfies multi-message multi-ciphertext pseudorandomness, then the ME construction satisfies multi-element pseudorandomness with consistency error $n/|M|$.

- SKE candidate: PRP-based construction such as AES $\rightsquigarrow$ compact ciphertext
- PKE candidate: unclear for the time being (deterministic PKE?)

Vector Oblivious Decrypt

Since the decryption result of ME is only 1-bit to indicate membership, thus the accompanying VODM can be simplified to VOD.



receiver

sender

$k/sk,\ m \in M$

VOD

$\mathbf{c} = (c_1, \ldots, c_n) \in C^n$

$\mathbf{e} = (e_1, \ldots, e_n) \in \{0,1\}^n$

$b_i = \text{Decrypt}(k/sk, c_i)$

Outline

1. Background
2. Starting Point: KRTW Protocol
3. Generic Construction of PSU
4. Two Instantiations of Generic Framework
5. Improvement and Optimization
6. Performance

Performance

n	Protocol	Comm. (MB)					Running time (s)															
		$\mathcal{R}$		$\mathcal{S}$		total	LAN				1Gbps				100Mbps				10Mbps			
		setup	online	setup	online		$T=1$		$T=8$		$T=1$		$T=8$		$T=1$		$T=8$		$T=1$		$T=8$	
							setup	online	setup	online	setup	online	setup	online	setup	online	setup	online	setup	online	setup	online
2^{14}	KRTW	0.02	4.17	0.01	29.63	33.8	0.07	3.5	0.03	1.07	0.49	16.13	0.37	14.06	0.83	27.36	0.72	24.66	0.81	55.9	0.73	55.32
	GMRSS	0.02	5.89	0.02	7.96	13.85	0.1	1.01	0.04	0.42	0.66	1.96	0.46	1.28	1	3.53	0.91	2.97	1.06	14.44	0.93	13.97
	JSZDG-R	0.01	4.65	0.01	5.63	10.28	0.07	1.81	0.02	0.52	0.27	2.65	0.23	1.34	0.49	4.19	0.41	2.66	0.45	12.08	0.37	10.63
	SKE-PSU	0.01	3.16	0	3.36	6.52	0.03	0.65	0.02	0.29	0.12	6.76	0.11	6.48	0.21	12.66	0.19	12.09	0.2	15.62	0.19	15.59
	PKE-PSU	0.01	1.16	0	1.59	2.75	4.6	2.37	4.58	1.07	4.78	2.63	4.75	1.34	4.92	3.02	4.9	1.77	4.99	4.43	4.91	3.79
	PKE-PSU*	0.01	2.16	0	2.9	5.05	4.6	1.96	4.6	0.59	4.75	2.36	4.76	1	4.95	2.76	4.91	1.54	4.92	5.72	4.93	5.31
2^{16}	KRTW	0.02	17.64	0.01	122.05	139.69	0.07	12.57	0.03	3.76	0.46	26.27	0.39	20.96	0.82	40.09	0.73	36.3	0.81	163.48	0.75	161.63
	GMRSS	0.02	25.95	0.02	34.11	60.06	0.11	4.79	0.04	1.95	0.64	6.61	0.48	4.25	1.11	12.67	0.92	9.78	1.04	60.75	0.94	57.5
	JSZDG-R	0.01	20.75	0.01	24.74	45.49	0.07	7.5	0.02	2.25	0.3	9.29	0.2	4.45	0.44	13.78	0.4	8.58	0.47	49.41	0.42	44.58
	SKE-PSU	0.01	12.61	0	13.41	26.03	0.04	2.66	0.02	1.15	0.13	8.66	0.11	7.32	0.2	15.84	0.19	14.39	0.2	31.79	0.19	30.98
	PKE-PSU	0.01	4.62	0	6.37	10.99	4.62	9.75	4.59	4.39	4.82	10.21	4.76	5.22	4.9	10.94	4.91	5.83	5.01	16.38	4.92	13.61
	PKE-PSU*	0.01	8.63	0	11.57	20.19	4.57	7.96	4.6	2.58	4.76	8.68	4.77	3.37	4.93	9.94	4.91	4.65	4.94	21.46	4.93	19.67
2^{18}	KRTW	0.02	69.29	0.01	562.76	632.05	0.08	63.02	0.03	17.67	0.52	85.56	0.39	45.31	0.76	111.14	0.71	113.83	0.84	660.33	0.74	664.93
	GMRSS	0.02	113.7	0.02	145.11	258.81	0.13	20.74	0.03	9.8	0.58	28.62	0.55	16.63	1.09	49.68	0.93	38.82	1.03	251.84	0.97	243.63
	JSZDG-R	0.01	92.67	0.01	107.89	200.56	0.07	41.15	0.03	10.71	0.25	43.17	0.21	16.84	0.42	64.06	0.4	33.8	0.53	221.27	0.39	191.2
	SKE-PSU	0.01	50.34	0	53.51	103.85	0.04	10.78	0.02	4.88	0.12	17.83	0.1	12.32	0.2	28.38	0.18	22.54	0.21	98.96	0.19	95.72
	PKE-PSU	0.01	18.5	0	25.45	43.95	4.6	41.5	4.59	19.82	4.79	42.37	4.75	20.97	4.92	44.8	4.91	23.38	4.92	66.68	4.9	54.39
	PKE-PSU*	0.01	34.5	0	46.26	80.76	4.61	34.63	4.58	12.26	4.78	37.1	4.75	13.99	4.92	40.62	4.92	18.45	4.91	85.31	4.92	79.22
2^{20}	KRTW	0.02	300.14	0.01	2305.8	2605.95	0.11	245.37	0.04	67.97	0.52	281.96	0.38	120.35	0.82	363.95	0.74	361.12	0.84	2643.84	0.75	2638.05
	GMRSS	0.02	493.2	0.02	615.9	1109.1	0.11	100.48	0.04	48.53	0.62	119.98	0.51	75.76	1.11	207.83	0.95	164.25	1.09	1074.33	0.95	1030.3
	JSZDG-R	0.01	405.53	0.01	467.26	872.79	0.08	173.07	0.04	54.41	0.48	184.63	0.2	73.28	0.47	266.51	0.73	146.13	0.47	941.5	0.72	825.16
	SKE-PSU	0.01	200.88	0	213.55	414.43	0.05	44.73	0.03	22.78	0.13	59.65	0.11	35.71	0.2	86.11	0.2	65.18	0.21	378.57	0.4	369.24
	PKE-PSU	0.01	74	0	101.8	175.8	4.65	168.79	4.6	79.95	4.78	169.18	4.79	86.49	4.97	179.58	4.94	96.32	4.97	269.32	4.87	216.19
	PKE-PSU*	0.01	138	0	185	323	4.64	144.24	4.58	50.56	4.75	146.41	4.74	60.5	4.9	161.26	5	76.33	4.99	345	4.9	313.37

- communication: $3.7 - 14.8\times$ reduction depending on set sizes
- running time: $1.2 - 12\times$ speed-up depending on network enviroments and set sizes

Thanks for Your Attention!

Any Questions?

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